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30 AUGUST 1967

MISSION AS-501 CM/SM SEPARATION
AND RECONTACT ANALYSIS
- ENTRY PHASE

By Mission Simulation Department
TRW Systems

MSC Task Monitor
R. E. McAdams



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MISSION PLANNING AND ANALYSIS DIVISION

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NAS 9-4810

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FOREWORD

This report presents an analysis of CM/SM separation and possible recontact for Apollo Mission AS-501. The report is submitted to the NASA/Manned Spacecraft Center as part of Task MSC/TRW A-122, Separation and Recontact Analysis for Apollo Missions, of the Apollo Mission Trajectory Control Program, under Contract No. NAS 9-4810.

The analysis includes all entry conditions possible for the nominal, aborted, and alternate missions. All classes of CM and SM trajectories are considered.

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NOMENCLATURE

		<u>Unit</u>
C_D	Drag coefficient	--
D	Drag force	lb
F	External force	lb
g	Nondimensional acceleration or ratio of acceleration to 32.2 ft/sec ²	--
h	Altitude	ft
L	Lift force	lb
m	Mass	slug
q	Dynamic pressure	lb/ft ²
r	Vector from earth center to c. g. of CM	ft
S	Reference area of CM and SM	ft ²
U_H	Components of inertial velocity as defined in Appendix	deg
V_H		
W_H		
V	Velocity	ft/sec
$\frac{W}{C_D S}$	Ballistic coefficient	lb/ft ²
X_F	Flight-path coordinates as defined in Appendix	--
Y_F		
Z_F		
X_H	Horizontal frame coordinates as defined in Appendix	--
Y_H		
Z_H		
β	Atmospheric density function	--
γ	Flight-path angle (positive above local horizontal)	deg

NOMENCLATURE (Continued)

		<u>Unit</u>
δ	Deviation from reference circular orbit	--
θ	True anomaly or central angle	deg
μ_H	Nondimensional horizontal velocity	--
ξ	Pitch attitude at separation	deg
ρ	Atmospheric density	slug/ft ³
$\delta\rho$	Nondimensional radius vector	--
τ	Nondimensional time	--
ϕ	CM roll angle	deg
ω_H	Nondimensional vertical velocity	--
ω_E	Earth rotation velocity	rad/sec

Subscripts:

0	Refers to reference circular orbit
1	Refers to CM
2	Refers to SM
A	With respect to atmosphere
F	Refers to flight-path coordinates
H	Refers to horizontal coordinate system
I	Inertial quantity

1. INTRODUCTION AND SUMMARY

An investigation of the CM/SM separation sequence and recontact possibilities for the entry phase of Mission AS-501 was conducted. The purpose of this mission is to verify the adequacy of the Block II heat shield when subjected to lunar return conditions. Lunar return is simulated by atmospheric entry from a high apogee, earth orbit, ellipse. Previous CM/SM separation and recontact studies have not included investigation of such high entry velocities and SM weights.

The method of analysis and recontact criterion employed for this analysis are essentially the same as that for the AS-204 analysis (Reference 1) which was performed under Task MSC/TRW A-3.

- Performance envelopes for CM trajectories were generated using the steepest and shallowest trajectories based on vehicle aerodynamics and guidance logic.
- All classes of SM motion during entry were considered:
 - Spin-stabilized
 - Tumbling
 - Trimmed
- CM/SM relative altitude and range were used to determine if recontact might occur.
- The criterion for reentry without recontact required that the CM performance envelope did not include the SM position.

The AS-501 nominal, aborted, and alternate missions were considered. Separation was assumed to occur at an attitude of 60 degrees above the inertial velocity vector when time-of-free-fall was 85 seconds.

The results of the study showed:

- The ballistic coefficient ratio of the two bodies was found to be the single most important parameter for assessing recontact during reentry.
- Recontact could not be completely ruled out when the ratio of SM to CM ballistic coefficients was between 0.88 and 1.16.

- Ballistic coefficient ratios where recontact cannot be completely ruled out occur during:
 - 1) Mode III Aborts
 - 2) Aborts during the second SPS burn
 - 3) Alternate missions resulting from contingencies during the second S-IVB burn
 - 4) The nominal mission
- No problems of recontact were present when the ratio of SM to CM ballistic coefficients was lower than 0.88 or higher than 1.16
- Separation attitude had only a small influence on the possibilities of recontact.

2. METHOD OF ANALYSIS (CM/SM SEPARATION)

2.1 COMPUTER PROGRAM

The equations of motion were solved on an analog computer using the Fogarty and Howe technique (Reference 2) which calculates the motion relative to a reference circular orbit. The equations employ three degrees-of-freedom in translation.

Details of the equations of motion are presented in the Appendix and a complete description of the analog program is given in Reference 3.

2.2 CRITERION FOR RECONTACT

The output of the computer program is presented as the time varying position of the CM with respect to the SM in terms of relative altitude and relative horizontal range. This output is plotted on an X-Y plotter with the SM always at the origin. A typical plot of computer output is shown in Figure 2-1. This output format was chosen because it can best represent the CM performance envelope.

The philosophy of generating a performance envelope for the CM guided trajectories has been retained from the Mission AS-204 analysis (Reference 1). This is a valuable tool because it allows bounding of the possible CM trajectories by the steepest and shallowest entry profiles (that is, the lowest and highest altitude profiles, respectively, as a function of time) allowed by vehicle aerodynamics and entry guidance logic. The shallowest profile is clearly a lift vector up trajectory from entry to impact. The steepest profile can be represented by a lift vector down trajectory that is controlled by modulating the lift vector in roll so that a resultant deceleration of 10 g is reached as quickly as possible and subsequently maintained.

Irregardless of the type of SM motion, its position is maintained at the origin. The criterion for no possible recontact is that the CM performance envelope does not enclose the SM position. Possible recontact is based on CM and SM motion within the orbital plane, since the extremities of vertical and down range positions are produced by the presence of the lift and drag vectors in the plane.

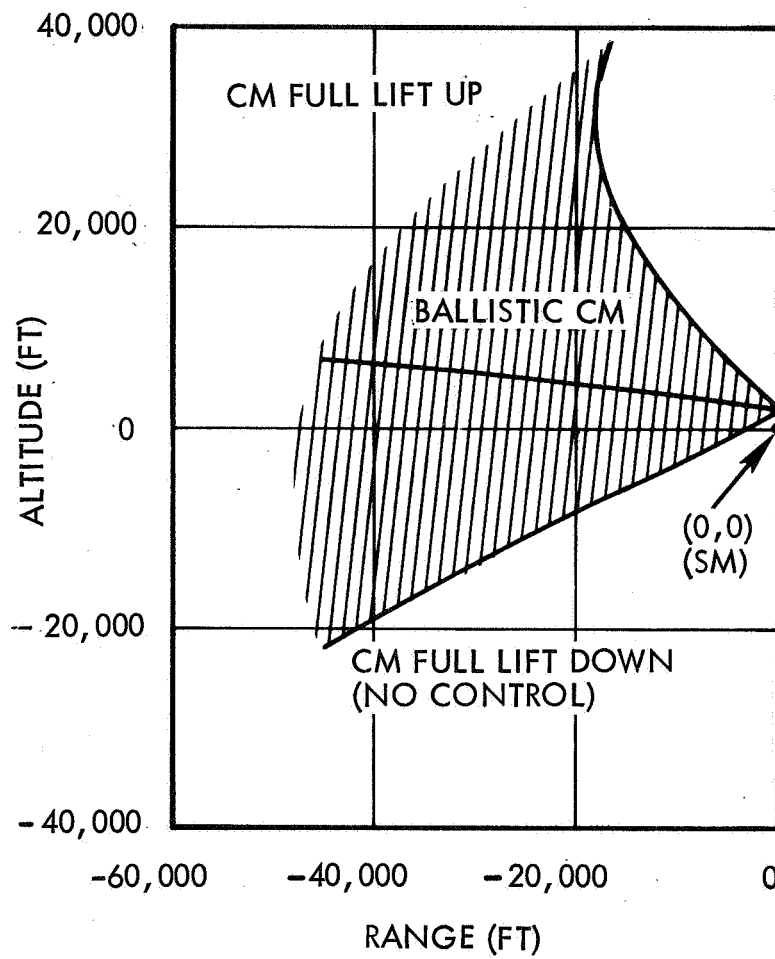


Figure 2-1. Typical Computer Program Output

2.3 ENTRY CONDITIONS INVESTIGATED

The objective of the AS-501 mission is to qualify the reentry heat shield under simulated lunar return velocities. In order to achieve this objective, the nominal mission plan requires four discrete burns; two of the S-IVB and two of the SPS. If an abort occurs during any one of these burns, a wide range of entry conditions (V_I and γ_I) can exist. These are given in Reference 4.

Similarly, if contingencies cause the alternate mission plan to be followed, a wide range of other entry conditions can exist. These are given in Reference 5.

The analysis of CM/SM separation must consider all of these possible entry conditions. They are presented in summary in Figure 2-2.

For all the entry conditions, various weights and ballistic coefficients of the SM are possible. These are based on data also found in References 4, 5, 6, and 7 and are outlined in Table 2-1.

2.4 AERODYNAMIC CHARACTERISTICS

The following aerodynamic characteristics were used in this analysis. They are taken from References 4, 6, and 8.

	<u>CM</u>	<u>SM</u>
Drag coefficient	1.182	1.8
Lift-to-drag ratio	0.38	0.3
Reference area (ft ²)	129.35	129.35
Weight (lb)	12,039	9590 - 39,810 (see Table 2-1)
Ballistic coefficient $\left(\frac{W}{C_{DS}}\right)\left(\frac{lb}{ft^2}\right)$	78.742	41.189 - 170.983 (see Table 2-1)

ENTRY ALTITUDE IS 400,000 FEET FOR ABORTS DURING SECOND S-IVB BURN, ABORTS DURING THE SPS BURNS, AND ALTERNATE MISSION ENTRIES.

ENTRY ALTITUDE IS 280,000 FEET FOR LAUNCH PHASE AND MODE II/MODE III ABORTS.

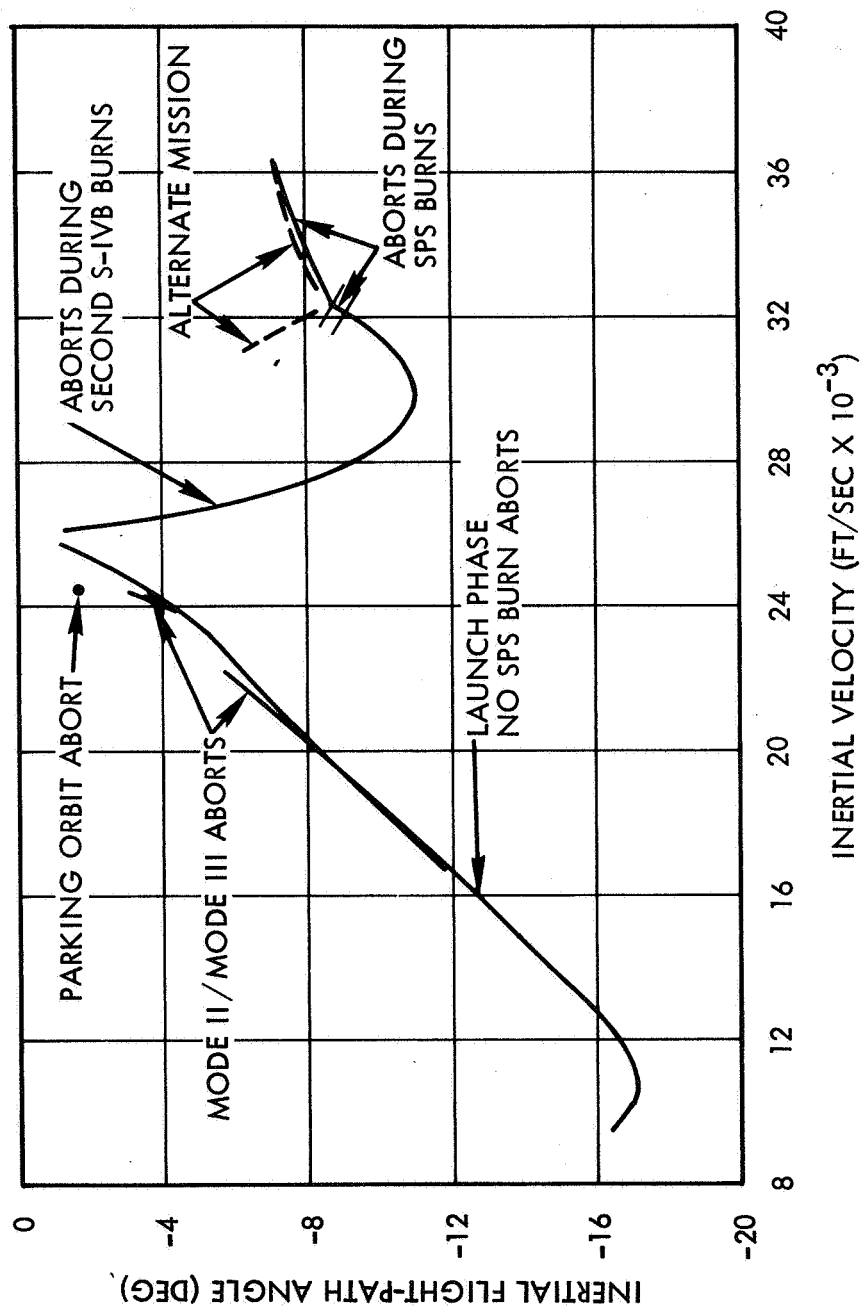


Figure 2-2. Summary of Possible Entry Conditions

Table 2-1. SM Weights and Ballistic Coefficients

<u>Trajectory</u>	<u>SM Weight (lb)</u>	$\left(\frac{W}{C_D S}\right)_{SM}$	$\left(\frac{W}{C_D S}\right)_{SM} / \left(\frac{W}{C_D S}\right)_{CM}$
<u>Aborts</u>			
Launch phase - no SPS burn	39, 810	170.983	2.171
Mode II/Mode III	20, 350 to 39, 810	87.403 to 170.983	1.110 to 2.171
Parking orbit	37, 050 or 38, 979	159.129 or 167.414	2.021 or 2.126
During second S-IVB burn	39, 810	170.983	2.171
During SPS burns	19, 660 to 39, 810	84.439 to 170.983	1.072 to 2.171
<u>Alternate Missions</u>			
During first S-IVB burn	9, 590 to 11, 010	41.189 to 47.288	0.523 to 0.601
From earth parking orbit	11, 010	47.288	0.601
During second S-IVB burn- without second SPS burn	11, 010 to 38, 550	47.288 to 165.571	0.601 to 2.103
During second S-IVB burn- with second SPS burn	9, 590 to 20, 010	41.189 to 85.943	0.523 to 1.091
No second S-IVB burn	9, 590	41.189	0.523
<u>Nominal Mission</u>	19, 660	84.439	1.072

3. RESULTS

3.1 ATTAINMENT OF SEPARATION DISTANCE BETWEEN CM AND SM

The CM/SM separation sequence is the same whether the mission is aborted, alternate, or nominal. Physical separation occurs when the time-of-free-fall to the entry interface is calculated to be 85 seconds. Entry interface is defined to be 400,000 feet altitude for all cases in this investigation with the exception of launch phase aborts. (Launch phase aborts include Mode II, Mode III, and no SPS burn aborts.) Entry interface for launch phase aborts is defined to be 280,000 feet. Separation between the CM and SM is caused by the firing of the SM RCS-X translation jets until propellant depletion. Sufficient propellant is available (Reference 7) to allow for RCS burning from separation until the RCS force is surpassed by the onset of aerodynamic forces (0.2 g).

Separation distance at the onset of aerodynamic deceleration will be dependent on both the time from separation and the SM weight. The time is almost entirely dependent on the entry flight-path angle. This effect can be seen in Figure 3-1. The dependence of separation distance on SM weight is shown in Figure 3-2. These distances are based on a low performance SM RCS (Reference 6).

3.2 SEPARATION ATTITUDE

CM/SM attitude at separation is 60 degrees above the inertial velocity vector. During the 85 seconds from separation until entry interface, a change in range angle of up to 8.2 degrees is generated. Additional range angle is generated for the times indicated in Figure 3-1 until the onset of aerodynamic deceleration. However, for all of the cases in this investigation, the SM is below and behind the CM at the onset of aerodynamic deceleration. This is illustrated in Figure 3-3.

3.3 RECONTACT POSSIBILITIES DURING TYPICAL ENTRY

The most significant parameter affecting possible recontact between the CM and the SM is the ballistic coefficient ratio of the two bodies. The value of this ratio during the various aborts and alternate missions of

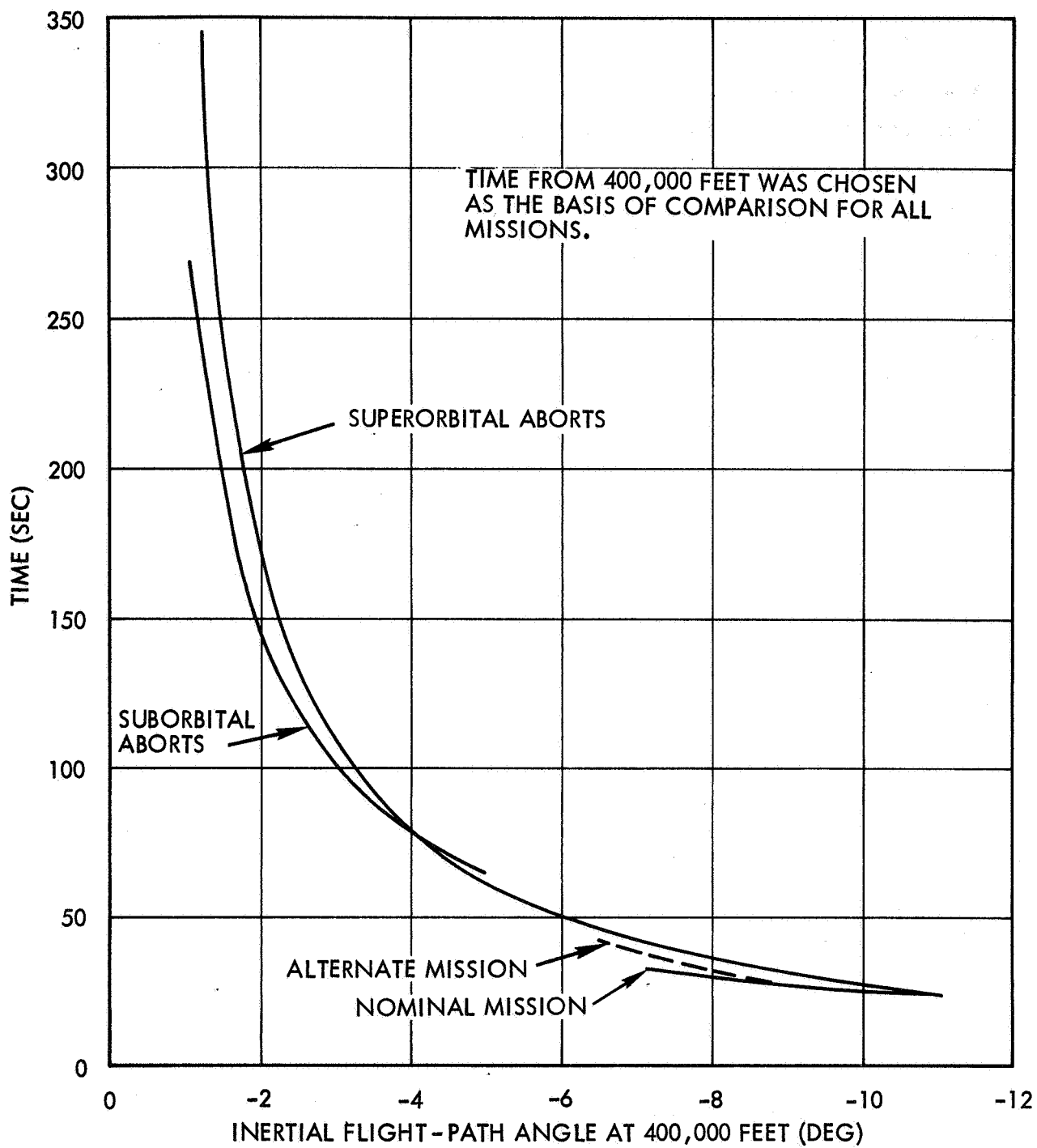


Figure 3-1. Times from 400,000 Feet to Onset of Aerodynamic Deceleration

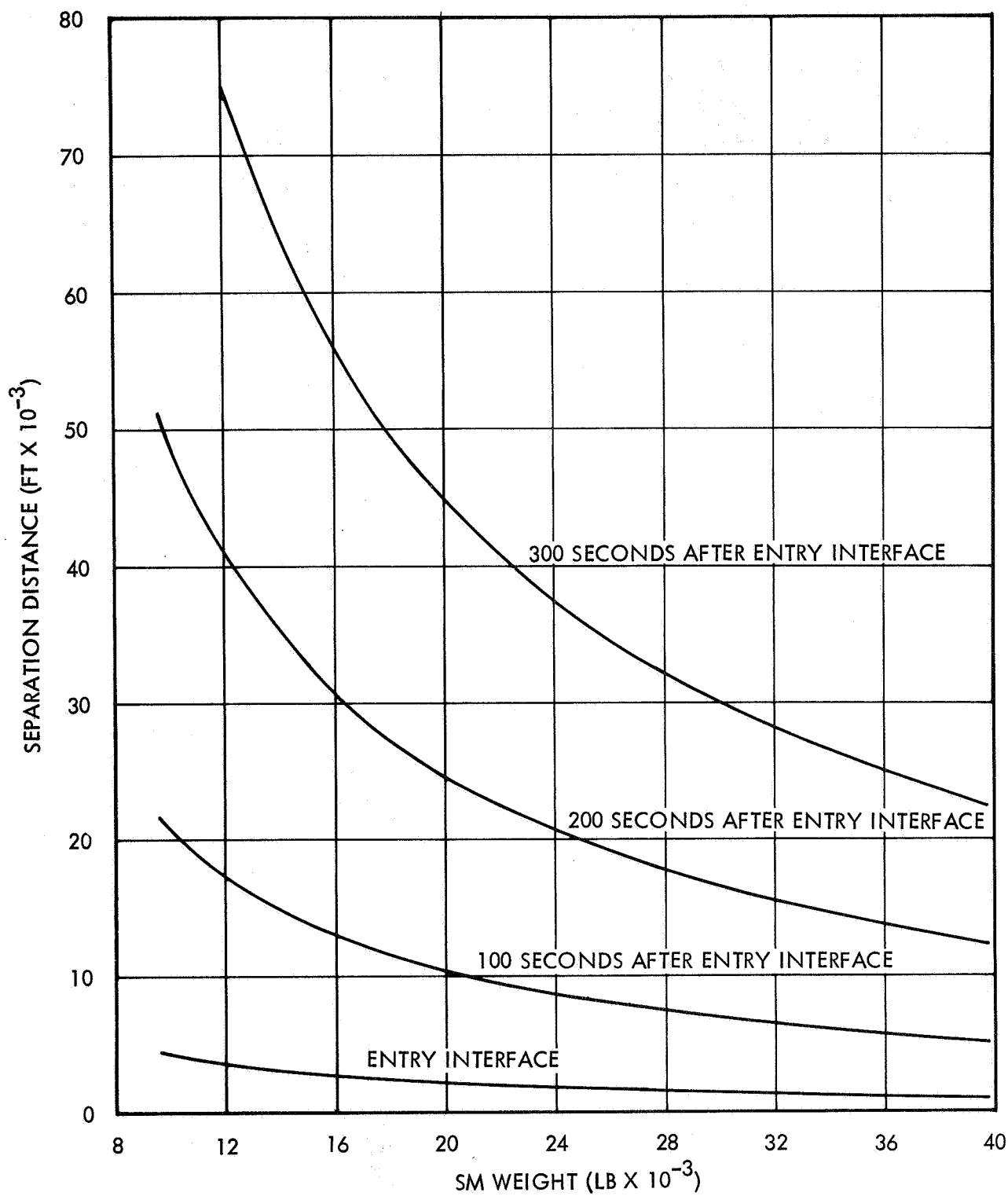


Figure 3-2. Separation Distances as a Function of SM Weight

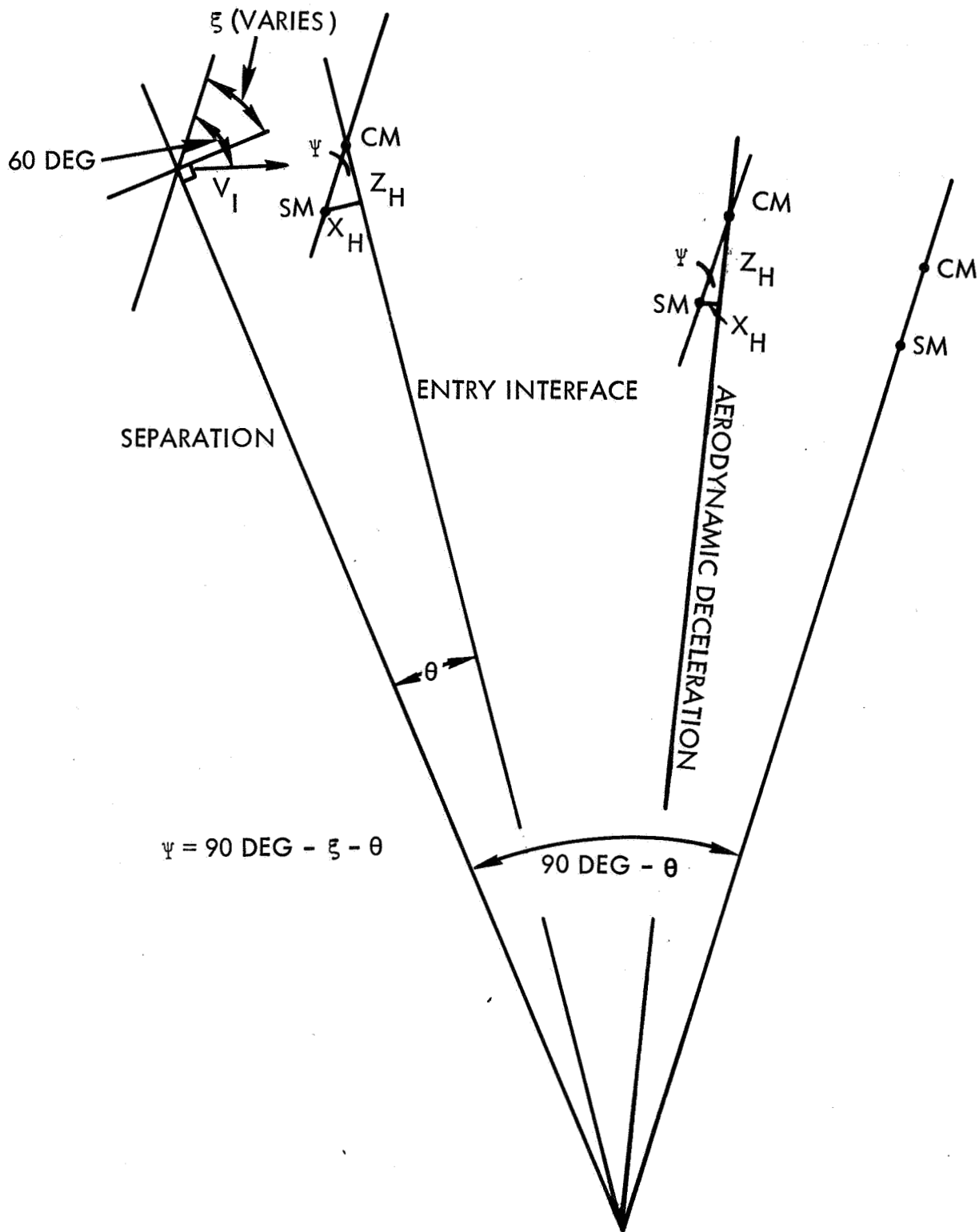


Figure 3-3. Effect of Separation Attitude

Mission AS-501 is shown in Figure 3-4. When expressed as $(W/C_D S)_{SM} / (W/C_D S)_{CM}$, this ratio becomes directly a function of SM weight.

For entry trajectories in which the ballistic coefficient ratio is greater than 1.16 or less than 0.88, there are no recontact problems. For these cases, the aerodynamic forces generate a horizontal range differential between the two bodies. This precludes any recontact during entry.

If the ballistic coefficient ratio is between 0.88 and 1.16, very little range differential is produced. The possibility of recontact then depends mainly on the SM trajectory. The spin-stabilized SM will either have a coning motion or tumble. (In either case, it can be represented aerodynamically by a point mass with the tumbling drag coefficient (Reference 10.)) This motion will cause the SM to fly lower than the entire CM performance envelope for entries included in this analysis. If the SM fails to spin up, it is possible for it to trim (Reference 8). Trimming with its lift vector up in the orbital plane can produce possible recontact problems for entry trajectories in the cross-hatched area of Figure 3-4. On the basis of this general discussion, specific entries can now be examined.

3.3.1 Launch Phase, No SPS Burn Aborts, Mode II Aborts, Parking Orbit Aborts, Aborts During the Second S-IVB Burn, and Aborts During the First SPS Burn

All of these aborts fall into the category in which the ballistic coefficient ratio is greater than 1.16. This results from the relatively small amount of SM propellant expenditure. Therefore, the SM flies ahead of the CM during entry trajectories due to any of these aborts.

3.3.2 Mode III Aborts

Early Mode III aborts involve long SPS burn times in an attempt to reach the discrete recovery area. This drives the ballistic coefficient ratio less than 1.16. The steepest CM trajectory for these cases is rolling, or effectively ballistic. But recontact with the trimmed SM cannot be ruled out if the SM lift vector is up in the orbital plane.

3.3.3 Aborts During the Second SPS Burn and Nominal Entry

These cases involve normal SPS propellant burning as a part of the nominal mission. However, during the latter stages of the SPS burn, the ballistic coefficient ratio is driven to less than 1.16. Since high entry velocities are present, a CM entering with its lift vector down rapidly reaches the 10 g limit. Therefore, the g-limiting loop in the CM guidance logic causes the lift vector to roll up to avoid excessive loads. This effect is shown in Figure 3-5. This allows spin-stabilized or tumbling SM's to maintain lower trajectories than the entire CM performance envelope. However, recontact with the trimmed SM with its lift vector up in the orbital plane cannot be completely ruled out.

3.3.4 Alternate Missions During the First S-IVB Burn or During Parking Orbit

These alternate missions involve large SPS expenditures in order to achieve the target ellipse. This drives the ballistic coefficient ratio to less than 0.88. Therefore, for all entry trajectories due to these alternate missions, the CM flies ahead of the SM, and recontact is no problem.

3.3.5 Alternate Missions During the Second S-IVB Burn

These alternate missions involve different amounts of SPS burning, but for some entry trajectories the ballistic coefficient ratio is between 0.88 and 1.16. This can be seen from Figure 3-4. These are all high speed entries, and hence, the arguments of Section 3.3.3 may be applied here. The conclusion, therefore, is that recontact with the trimmed SM with its lift vector up in the orbital plane cannot be completely ruled out.

3.4 OPTIMUM SEPARATION ATTITUDE

Separation attitude is not a critical parameter in determining possible recontact problems. This is in general agreement with the Mission AS-204 study (Reference 1). The separation attitude for Mission AS-501 performs the function of placing the SM almost directly below the CM at the onset of aerodynamic deceleration as noted in Section 3.2. In Figure 3-6, this would be along line OA. However, if as a result of a change in separation attitude, CM and SM relative positions were along lines OB or OC, there

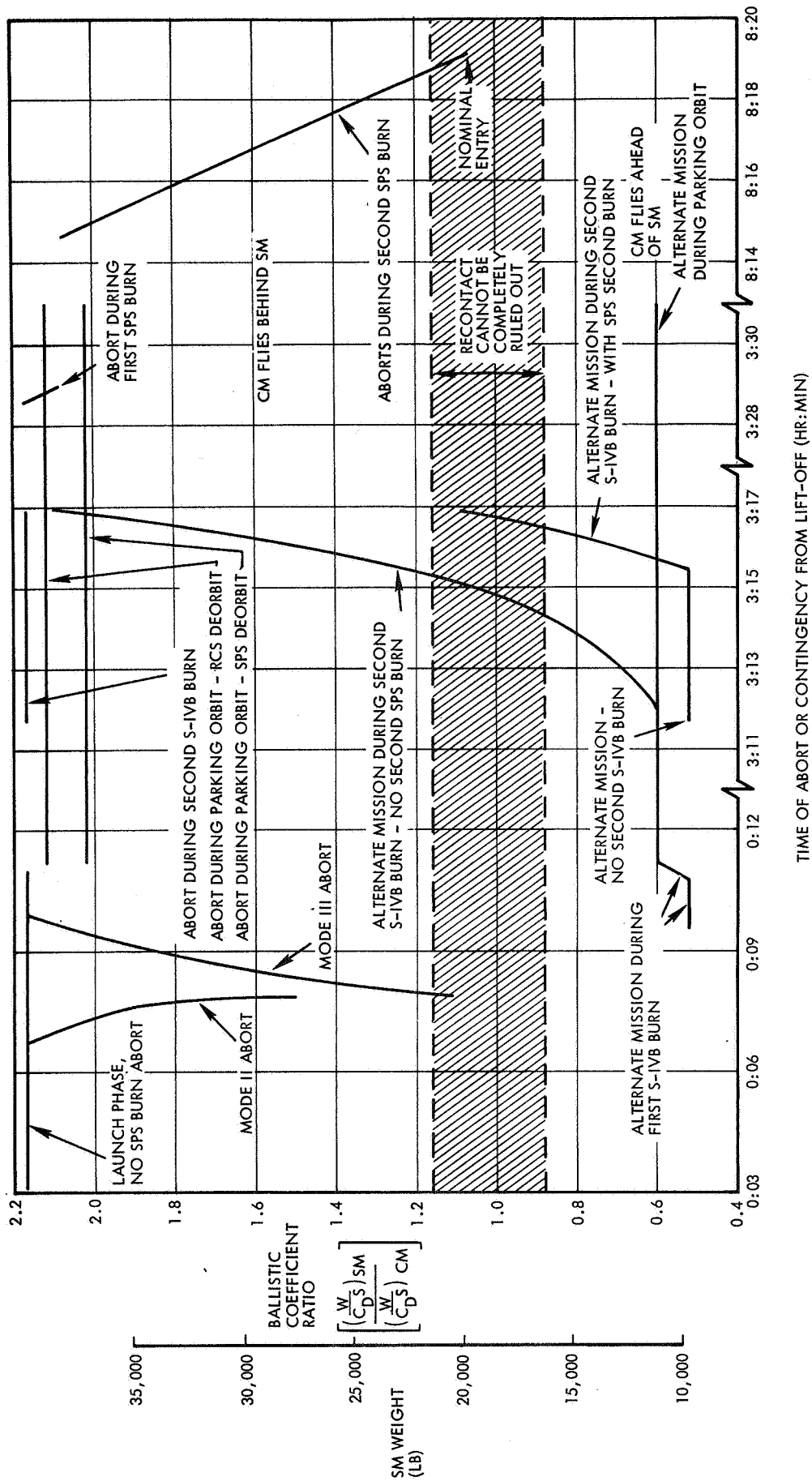


Figure 3-4. Effect of Ballistic Coefficient Ratio

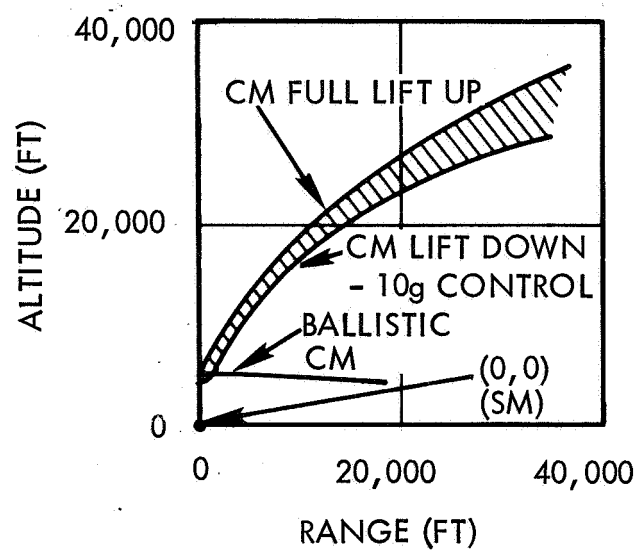


Figure 3-5. Example of g-limited CM Trajectory

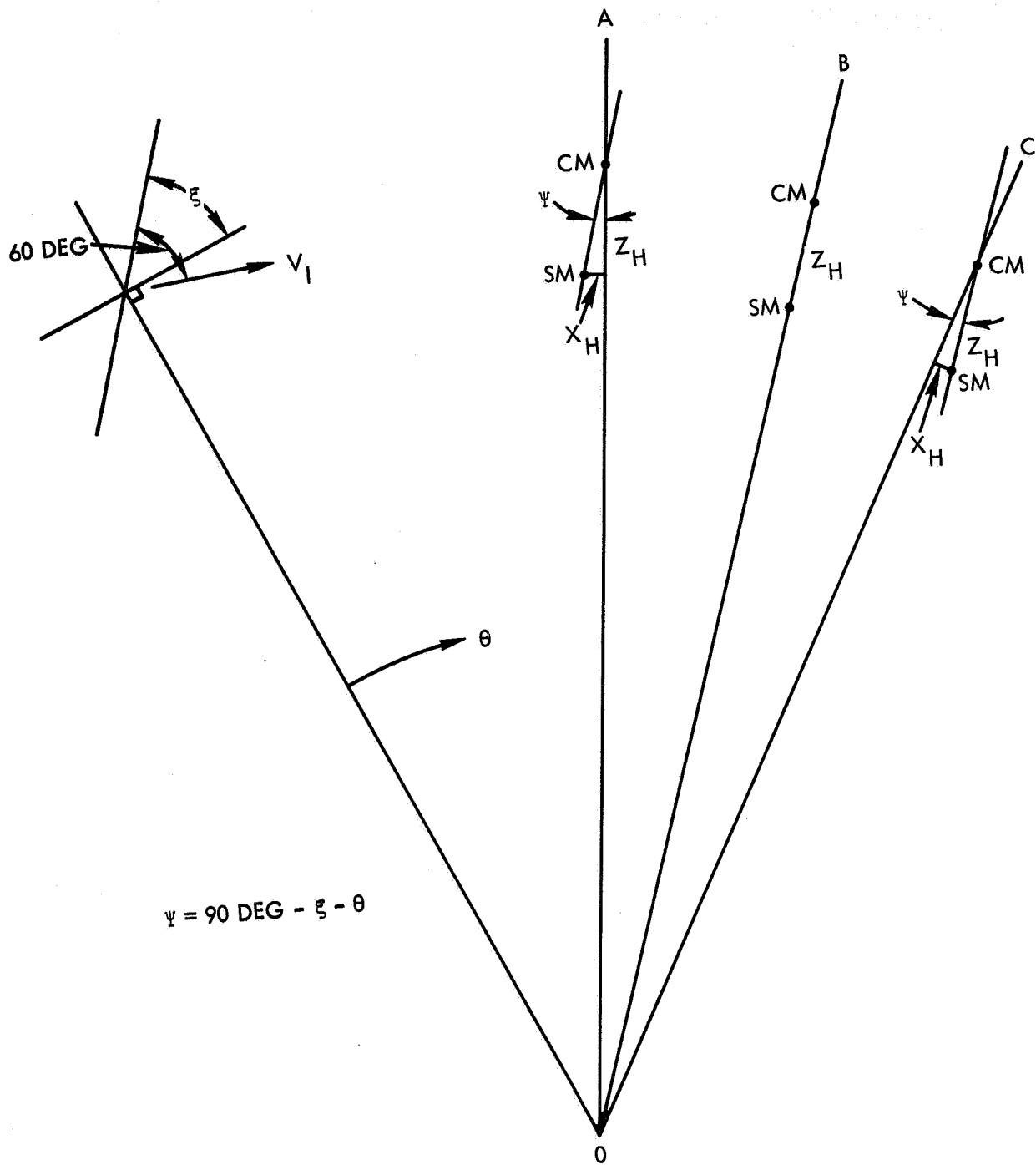


Figure 3-6. Optimum Separation Attitude

would be only small changes in the discussion of recontact problems heretofore presented. The effect of separation attitude is far surpassed by the effect of ballistic coefficient ratio.

4. CONCLUSIONS

The Mission AS-501 CM/SM separation sequence (separating at an attitude of 60 degrees above the inertial velocity vector when time-of-free-fall is 85 seconds) has been evaluated for entry trajectories due to the nominal, aborted, and alternate missions. The most significant parameter affecting possible recontact has been found to be the ballistic coefficient ratio of the two bodies.

Ballistic coefficient ratios of close to unity occur during the nominal mission, aborts (Mode III and during the second SPS burn), and for alternate missions (resulting from contingencies during the second S-IVB burn). For all of these cases, recontact of the CM with a trimmed SM with its lift vector up in the orbital plane cannot be completely ruled out. However, spin-stabilized or tumbling SM's will always have lower trajectories than those of the CM for these cases.

Entry trajectories caused by all of the other aborts and alternate missions involve ballistic coefficient ratios considerably greater than or less than unity. This generates a range differential between the two bodies and precludes any recontact.

Separation attitude has only a small influence on the possibilities of recontact.

APPENDIX

CM/SM SEPARATION MATHEMATICAL MODEL

This mathematical model represents an updating of the model found in Reference 1. The principal improvement has been the removal of small angle approximations.

1. COORDINATE SYSTEM DEFINITION

The equations are developed in terms of a rotating coordinate system fixed to the CM center of gravity. This system is termed the horizontal reference frame and is denoted by X_H , Y_H , and Z_H . The Z_H axis is defined along the local vertical, positive in a direction towards the earth center, as shown in Figure A-1. A right-handed system is formed by X_H (the local horizontal) and Y_H with rotation of the system occurring about Y_H caused by the change in true anomaly, θ . The components of the inertial velocity vector in the horizontal reference frame are called U_H , V_H , and W_H , respectively. The flight-path coordinates (X_F , Y_F , and Z_F) are defined so that X_F is aligned along the air velocity vector, V_A . These coordinates are formed by rotating the horizontal reference frame about Y_H through an angle, $-\gamma_A$. Drag, D , and lift, L , are defined as positive in the $-X_F$ (back) and $-Z_F$ (up) directions, respectively. CM lift vector orientation (roll angle) is defined by rotation about X_F .

2. EQUATIONS OF MOTION

The basic equations of motion are used in a form developed by Fogarty and Howe (Reference 1) which minimizes computer scaling problems. These equations are written about a reference circular orbit and are given as:

$$\delta\mu_H = \frac{\delta U_H}{U_{H0}} = \frac{1}{1+\delta\rho} \left[\int (1+\delta\rho) \frac{F_{X_H}}{mg_0} d\tau - \delta\rho + (\delta\rho + \delta\mu_H + \delta\rho\delta\mu_H)_{t=0} \right] \quad (1)$$

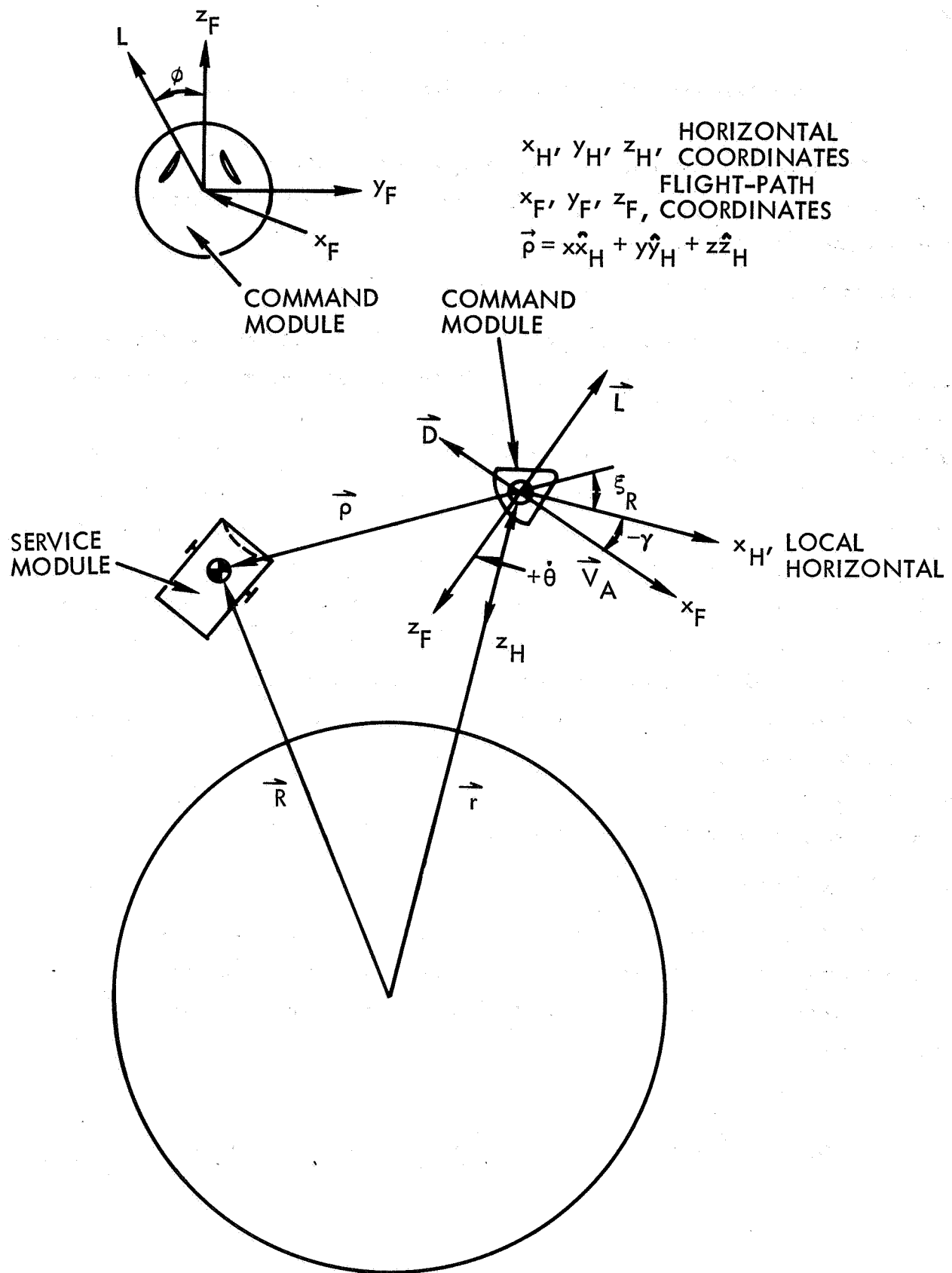


Figure A-1. Definition of Coordinate System

$$\frac{d\omega_H}{d\tau} = -\frac{d^2\delta\rho}{d\tau^2} = -\frac{\delta\rho}{(1+\delta\rho)^2} - \frac{2\delta\mu_H + (\delta\mu_H)^2}{1+\delta\rho} + \frac{F_{ZH}}{mg_0} \quad (2)$$

The nondimensional parameters in Equations (1) and (2) are defined as follows:

$$\delta\rho = \frac{\delta r}{r_0}, \quad \omega_H = \frac{W_H}{U_{H_0}}, \quad \tau = \sqrt{\frac{g_0}{r_0}} t, \quad \delta\mu_H = \frac{\delta U_H}{U_{H_0}} \quad (3)$$

Subscript zero refers to the reference circular orbit that was defined at an altitude of 400,000 feet in this study. The corresponding circular orbit velocity was taken as 25,700 feet per second at a g level of 31 feet per second². Deviations from the circular orbit are indicated by δ preceding the variable.

The aerodynamic forces are resolved from the flight-path angle coordinates to the horizontal system by the following transformation.

$$\begin{bmatrix} F_{XH} \\ F_{YH} \\ F_{ZH} \end{bmatrix} = \begin{bmatrix} -\cos \gamma_A & 0 & -\sin \gamma_A \\ 0 & -1 & 0 \\ \sin \gamma_A & 0 & -\cos \gamma_A \end{bmatrix} \begin{bmatrix} D \\ L \sin \phi \\ L \cos \phi \end{bmatrix} \quad (4)$$

The dynamic pressure computation for the CM is based on a modified exponential atmosphere model.

$$q = 1/2 \rho_0 V_A^2 e^{-\beta(h) h} \quad (5)$$

The variation of β with altitude was obtained by fitting the density expression $\rho = \rho_0 e^{-\beta(h) h}$ to the 1962 ARDC atmosphere. Because of the problems associated with computing exponential functions on the analog computer, the equation was nondimensionalized and put into log form as follows:

$$\frac{Sq}{m} = \frac{S \rho_0 V_A^2 e^{-\beta(h) h}}{2m}$$

Since

$$U_{H_0} = \sqrt{r_0 g_0} \quad (6)$$

$$\frac{S q}{m g_0} = \frac{S r_0 \rho_0 V_A^2 e^{-\beta(h) h}}{2 m U_{H_0}^2}$$

$$\ln \frac{S q}{m g_0} = \ln \frac{S r_0 \rho_0}{2 m} + 2 \ln \frac{V_A}{U_{H_0}} - \beta(h) h \quad (7)$$

The dynamic pressure on the SM is computed as a function of the dynamic pressure on the CM.

$$q_2 = (\delta q) q_1 \quad (8)$$

The nondimensional $\ln \delta q$ is also solved for

$$\begin{aligned} \ln \delta q &= \ln \frac{q_2}{q_1} \\ &= \ln \frac{V_{A_2}^2 e^{-\beta(h_2) h_2}}{V_{A_1}^2 e^{-\beta(h_1) h_1}} \\ &= \ln \left(\frac{V_{A_2}}{V_{A_1}} \right)^2 e^{\beta(h_1) h_1 - \beta(h_2) h_2} \\ &= 2 \ln V_{A_2} - 2 \ln V_{A_1} + \beta(h_1) h_1 - \beta(h_2) h_2 \end{aligned} \quad (9)$$

CM body acceleration in g's resulting from D_1 equals

$$\frac{D_1}{g_0} = \frac{C_{D_1} S_1 q_1}{m_1 g_0} \quad (10)$$

Equation (8) then becomes

$$q_2 = \delta q \left(\frac{D_1}{g_0} \right) \left(\frac{m_1 g_0}{C_{D_1} S_1} \right)$$

SM acceleration resulting from D_2 , therefore equals

$$\begin{aligned} \frac{D_2}{g_0} &= \frac{C_{D_2} S_2 q_2}{m_2 g_0} \\ &= \delta q \left(\frac{D_1}{g_0} \right) \left[\frac{\left(\frac{W}{C_D S} \right)_1}{\left(\frac{W}{C_D S} \right)_2} \right] \end{aligned} \quad (11)$$

The motion of the SM is computed with respect to the CM in terms of the horizontal coordinate frame. The relative equations of motion are derived in Reference 9 and are given as follows:

$$\ddot{X}_H = \frac{F_{X_2}}{m_2} - \frac{F_{X_1}}{m_1} - 2 \dot{\theta} \dot{Z}_H - \ddot{\theta} Z_H + \dot{\theta}^2 X_H \quad (12)$$

$$\ddot{Y}_H = \frac{F_{Y_2}}{m_2} - \frac{F_{Y_1}}{m_1} \quad (13)$$

$$\ddot{Z}_H = \frac{F_{Z_2}}{m_2} - \frac{F_{Z_1}}{m_1} + 2 \dot{\theta} \dot{X}_H + \ddot{\theta} X_H + \dot{\theta}^2 Z_H \quad (14)$$

X_H , Y_H , Z_H are the components of the separation vector, $\bar{\rho}$, along the horizontal coordinate system axes, and θ is the central angle which is initialized at 400,000 feet.

Several equations of constraint are required and are given below:

a) True anomaly rate

$$\dot{\theta} = - \frac{V_I \cos \gamma_I}{r} = - \frac{U_H}{r} \quad (15)$$

Using Equation (3), Equation (15) can be written in nondimensional form as

$$\theta = \int \frac{1 + \delta\mu_H}{1 + \delta\rho} d\tau \quad (16)$$

b) Relative velocity (zero degree posigrade inclination)

$$V_A = \left[(U_H - \omega_E \times r)^2 + W_H^2 \right]^{1/2} \quad (17)$$

c) Relative flight-path angle

$$\sin \gamma_A = \frac{W_H}{V_A} \quad (18)$$

d) SM RCS thrust computation

$$\ddot{X} = - \frac{F}{m} \Big|_{RCS} \cos \xi_R \quad (19)$$

$$\ddot{Z} = \frac{F}{m} \Big|_{RCS} \sin \xi_R \quad (20)$$

$$\xi_R = \xi + \theta \quad (21)$$

REFERENCES

1. R. H. Hoh and A. B. Simmons, "Mission AS-204 CM/SM Separation Prior to Reentry, " TRW 05952-6049-R0-00, 10 October 1966.
2. L. E. Fogarty and R. M. Howe, "Flight Simulation of Orbital and Reentry Vehicles, " IRE Transactions on Electronic Computers, Vol. EC-11, August 1962.
3. Charles L. Smith and Don R. Hayden, "Vehicle Separation Study During Reentry, " LEC 672-22-09007, August 1967.
4. "Spacecraft Operational Abort and Alternate Mission Plan for AS-501, Volume I-Abort Plan, " MSC Internal Note 66-FM-149, TRW 05952-H111-R0-00, 22 December 1966.
5. "Spacecraft Operational Abort and Alternate Mission Plan for AS-501, Volume II-Alternate Mission Plan, " MSC Internal Note 66-FM-149, TRW 05952-H142-R0-00, 26 January 1967.
6. "Apollo Mission Data Specification D, Apollo Saturn 501 (U), " TRW 2131-H009-R8-000, (C). *
7. D. M. Schneider, "AS-501 Spacecraft Operational Trajectory, Volume I, Trajectory Description, " TRW 05952-H067-R0-00, 15 November 1966.
8. E. D. Murrah, "Apollo Service Module Entry Aerodynamics, " MSC Memorandum 67-FM13-183, 16 May 1967.
9. R. H. Hoh, "The Equations of Motion for a Hybrid Computer Simulation of the General Separation Problem, " TRW IOC 9881.3-278, 16 August 1965.
10. E. J. Der, et al., "Summary Report of Apollo Separation and Recontact Analysis (U), " TRW 3869-6004-R8-000, 31 December 1965 (C).

*Includes numerous revisions which are normally incorporated with the report covers.